Name: ______________________________

Instructor: __________________________

Math 10560, Exam 3
April 22, 2014

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- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
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3.	(a)	(b)	(c)	(d)	(e)
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5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
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Multiple Choice	________
11.	________
12.	________
13.	________
Total	________

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Multiple Choice

1.(6 pts) Find the sum of the following series

$$\sum_{n=1}^{\infty}\left[\frac{\ln(n+1)}{n+2}-\frac{\ln(n+2)}{n+3}\right].$$

(a) $\frac{\ln(2)}{3}$ (b) This series diverges (c) $\frac{\ln(2)}{3}-\frac{\ln(3)}{4}$

(d) $\frac{\ln(2)}{3}-2$ (e) $\frac{\ln(2)}{3}-1$

2.(6 pts) Use the comparison test or limit comparison test to determine which of the following series are convergent:

$$(I)\quad \sum_{n=2}^{\infty}\frac{\sin^2(n)+1}{2\sqrt{n}} \qquad (II)\quad \sum_{n=2}^{\infty}\frac{n^2+2n+1}{n^4+2n^2+1} \qquad (III)\quad \sum_{n=1}^{\infty}\frac{1}{n2^n}$$

Which of the following statements is true?

(a) Only I and II converge (b) All three converge

(c) Only II converges (d) Only II and III converge

(e) All three diverge

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3.(6 pts) Consider the following series

$$(I)\quad \sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n}} \qquad (II)\quad \sum_{n=2}^{\infty} \frac{n}{\ln(n^2)} \qquad (III)\quad \sum_{n=1}^{\infty} \frac{3^{n+1}}{2(n!)}$$

Which of the following statements is true?

(a) Only I and II converge

(b) Only III converges

(c) All three converge

(d) All three diverge

(e) Only I and III converge

4.(6 pts) Consider the following series

$$(I)\quad \sum_{n=1}^{\infty} \frac{(n+1)!}{n^2 \cdot e^n} \qquad (II)\quad \sum_{n=1}^{\infty} \left(\frac{2^{n+1}}{2^n+1}\right)^n.$$

Which of the following statements is true?

(a) They both diverge.

(b) They both converge.

(c) (I) converges and (II) diverges.

(d) (I) diverges and (II) converges.

(e) Deciding whether these series converge or diverge is beyond the scope of the methods taught in this course.

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5.(6 pts) Which **one** of the following series converges conditionally?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2(n)}{3^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{5^n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n e^n}{e^n+1}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{n^5+1}$

6.(6 pts) Find a power series representation for the function

$$\frac{x^2}{(1-x^3)^2}$$

in the interval $(-1,1)$.
(Hint: Differentiation of power series may help).

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n 3nx^{3n-1}$ (b) $\displaystyle\sum_{n=1}^{\infty} nx^{n-1}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{x^{3n+1}}{3n+1}$

(d) $\displaystyle\sum_{n=1}^{\infty} nx^{3n-1}$ (e) $\displaystyle\sum_{n=1}^{\infty} x^{3n}$

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7.(6 pts) Use you knowledge of a well known power series to calculate the limit

$$\lim_{x \to 0} \frac{2\cos(x^2) - 2 + x^4}{x^8}$$

(a) $\frac{2}{8!}$

(b) $\frac{1}{12}$

(c) $\frac{1}{2}$

(d) The limit does not exist

(e) 2

8.(6 pts) Which of the following is the third Taylor polynomial of the function

$$f(x) = \sin\left(\frac{x}{2}\right) \text{ centered at } a = \pi?$$

(a) $1 - \frac{x^2}{4(2!)}$

(b) $1 - \frac{\left(x - \frac{\pi}{2}\right)^2}{4(2!)}$

(c) $1 - \frac{(x-\pi)^2}{4(2!)}$

(d) $(x-\pi) - \frac{(x-\pi)^3}{3!}$

(e) $(x-\pi) - \frac{(x-\pi)^3}{2}$

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9.(6 pts) Compute the radius of convergence, R, of the following power series

$$\sum_{n=0}^{\infty} \frac{x^n}{2^n(n+1)^2}$$

(a) $R = 2$ (b) $R = 5$ (c) $R = 1$

(d) $R = 1/2$ (e) $R = \infty$

10.(6 pts) Which of the following gives a power series representation of the function

$$f(x) = e^{-\frac{x^2}{2}}$$

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+2}}{2^n n!}$ (b) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+2}}{2^n (2n)!}$

(c) $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$ (d) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^n n!}$

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Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(10 pts.) (a) Consider the series $\sum_{n=3}^{\infty}(-1)^n\frac{(\ln n)^2}{n}$. Fill in the following blanks and be sure to **show your work**. In each case indicate which test you are using and show how it is applied.

- Is the series absolutely convergent? (**YES** or **NO**) ____________

- Is the series convergent? (**YES** or **NO**) ____________

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12.(15 pts.) (a) Give the Taylor series expansion for the antiderivative

$$F(x) = \int \cos\left(\sqrt{x}\right)\, dx$$

about 0 (McLaurin Series) where $F(0) = 0$.
Hint: Use your knowledge of a well known series.

(b) Use part (a) to find an expression for the definite integral

$$\int_0^1 \cos(\sqrt{x})\, dx$$

as a sum of an infinite series.

(c) Use the alternating series estimation theorem to estimate the value of the above definite integral so that the error of estimation is less than $\frac{1}{100}$.
(you may write your answer as a sum of fractions).

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13. (15 pts.) Find the radius of convergence and interval of convergence of the following power series:

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x+3)^n}{5^n \sqrt{n+1}}.$$

R.O.C.____________ **I.O.C.**____________

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The following is the list of useful trigonometric formulas:

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}\big(\sin(x - y) + \sin(x + y)\big)$$

$$\sin x \sin y = \frac{1}{2}\big(\cos(x - y) - \cos(x + y)\big)$$

$$\cos x \cos y = \frac{1}{2}\big(\cos(x - y) + \cos(x + y)\big)$$

$$\int \sec\theta = \ln|\sec\theta + \tan\theta| + C$$

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